CS 516—Software Foundations via Formal Languages—Spring 2022

Final Examination

Thursday, May 12, 12noon–2pm

Question 1 (10 points)

Let **RegLan**, **CFLan**, **RecLan** and **RELan** be, as usual, the sets of all regular, context-free, recursive and recursively enumerable languages. Fill in each cell of the following table with a "Y" (for "yes") or "N" (for "no") to indicate what closure properties these four sets of languages have:

	union	concatenation	closure	intersection	difference
RegLan					
CFLan					
RecLan					
RELan					

E.g., at the intersection of the row **CFLan** and column "difference", put a "Y" if you think the context-free languages are closed under set difference ($L_1 - L_2$), and a "N" if you think the context-free languages are not closed under set difference. (*When grading, one point will be deducted for each mistake.*)

Question 2 (20 points)

Let

$$X = \{\, \mathsf{0}^i \mathsf{1}^j \mathsf{2}^k \mid i, j, k \in \mathbb{N} \text{ and } (i = j \text{ or } j = k) \,\}.$$

Find a grammar G such that $L(G) = X$.

Question 3 (10 points)

Prove or disprove the following statement:

For all languages A and B,

$$(A \cap B)^* = A^* \cap B^*.$$

(If you think it is true, prove it. If you think it is false, find languages A and B so the equation doesn't hold, and show it doesn't hold.)

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Question 4 (15 points)

Suppose x is a string, Y is a set of strings, and $x \notin Y$. Suppose we have a DFA M such that $L(M) = Y \cup \{x\}$. Explain how we can turn M into a DFA N such that $L(N) = Y$. Explain why your answer is correct.

Question 5 (30 points)

Let

$$X = \{\, w \in \{0,1\}^* \mid 00 \text{ is not a substring of } w \,\}.$$

Thus, for all $w \in \{0,1\}^*$:

- $w \in X$ iff 00 is not a substring of w;
- $w \notin X$ iff 00 is a substring of w.

Find a DFA M such that $L(M) = X$, and—using our standard approach, involving induction on Λ—prove that your answer is correct.

You should make your proof as complete and clear as you can. One way to do this is to refer to parts of the following lemma (which you *don't* have to prove):

Lemma 5.1
(1) $\% \in X$.

(2) For all $w \in \{0,1\}^*$, *if* $w \in X$, *then* $w1 \in X$.

(3) For all $w \in \{0,1\}^*$, *if* $w \in X$ *and* 0 *is not a suffix of* w, *then* $w0 \in X$.

(4) For all $w \in \{0,1\}^*$, *if* 0 *is a suffix of* w, *then* $w0 \notin X$.

(5) For all $w \in \{0,1\}^*$ *and* $a \in \{0,1\}$, *if* $w \notin X$, *then* $wa \notin X$.

Question 6 (15 points)

Let

$$L = \{\, 0^i 1^j 2^k 3^l \mid i, j, k, l \in \mathbb{N} \text{ and } i < l \text{ and } j > k \text{ and } i + j \text{ is even and } k + l \text{ is odd } \}.$$

Prove that L is not regular.