CS 516—Software Foundations via Formal Languages—Spring 2022

Problem Set 3

Due by 5pm on Friday, March 4
Submission via Gradescope

Problem 1 (15 Points)

Use Forlan to carefully compare and contrast how local and global simplification work on the regular expression $(\mathsf{00}^*\mathsf{11}^*)^*$. In particular, identify the locally optimal reduction in local simplification that results in the final result being less simple than the result of global simplification. Include a transcript of your Forlan session as part of your submission on Gradescope. (You don't need to submit anything via GitHub.)

Problem 2 (30 Points)

Consider reduction rule (14) from Section 3.3.3 of the book:

> If $\mathbf{not}(\mathbf{hasEmp}\,\alpha)$ and $\mathbf{cc}\,\alpha \cup \overline{\mathbf{cc}\,\beta} <_{cc} \overline{\overline{\mathbf{cc}\,\beta}}$,
> then $(\alpha\beta^*)^* \to \% + \alpha(\alpha+\beta)^*$.

(a) What is the rationale for the rule only being applicable when $\mathbf{not}(\mathbf{hasEmp}\,\alpha)$? Hint: the rule is valid and reduces closure complexity without this restriction, so the rationale has to do with this rule's relationship to the other rules. [5 points]

(b) Prove that, for all $\alpha, \beta \in \mathbf{Reg}$, if $\mathbf{cc}\,\alpha \cup \overline{\mathbf{cc}\,\beta} <_{cc} \overline{\overline{\mathbf{cc}\,\beta}}$, then $\mathbf{cc}(\% + \alpha(\alpha+\beta)^*) <_{cc} \mathbf{cc}((\alpha\beta^*)^*)$. [25 points]

Problem 3 (55 points)

Let $X = \{\, w \in \{\mathsf{0},\mathsf{1}\}^* \mid \mathsf{010} \text{ is not a substring of } w \,\}$.

(a) Find a regular expression α such that $L(\alpha) = X$. [15 points]

(b) Prove that your answer to Part (a) is correct. [40 points]