

## Problem Set 4

Due by 5pm on Friday, March 25  
Submission via Gradescope and GitHub

### Problem 1 (35 points)

Let  $X = \{w \in \{0,1\}^* \mid 010 \text{ is not a substring of } w\}$ .

(a) Find and draw (e.g., using JForlan) a finite automaton  $N$  such that  $L(N) = X$ .  
[20 points]

(b) Define an SML/Forlan function

```
val test = fn : int -> fa -> str option * str option
```

such that, for all  $n \in \mathbb{N}$ , `test n` returns a function  $f$  such that, for all FAs  $N$ ,  $f N$  returns a pair  $(xOpt, yOpt)$  such that:

- If there is an element of  $\{0,1\}^*$  of length no more than  $n$  that is in  $X$  but is not accepted by  $N$ , then  $xOpt = \text{SOME } x$  for some such  $x$ ; otherwise,  $xOpt = \text{NONE}$ .
- If there is an element of  $\{0,1\}^*$  of length no more than  $n$  that is not in  $X$  but is accepted by  $N$ , then  $yOpt = \text{SOME } y$  for some such  $y$ ; otherwise,  $yOpt = \text{NONE}$ .

Use `test` to test your FA  $N$  from part (a) on all elements of  $\{0,1\}^*$  of length no more than 10. Include a transcript of your Forlan session in your PDF submission.

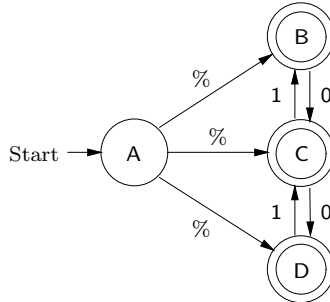
In the subdirectory `CS516-PS4` of your private GitHub repository you should put:

- a file `ps4-p1-fa` containing the expression in Forlan's syntax of  $N$ ;
- a file `ps4-p1.sml` consisting of the definition of `test`.

Hint: you may adapt code from the file `ps4-p2.sml` from Problem 2 of the old Problem Set 4.  
[15 points]

## Problem 2 (20 points)

Let  $M$  be the finite automaton



(a) Use Forlan to find and display labeled paths showing why the following three strings are accepted by  $M$ : 0010110, 1001101 and 1011001. Include a transcript of your Forlan session. [10 points]

(b) Define an SML/Forlan function

```
val accLen : int -> str set
```

such that for all  $n \in \mathbb{N}$ , `accLen n` returns the set of all strings  $w$  of length  $n$  over the alphabet  $\{0,1\}^*$  such that  $w$  is accepted by  $M$ . Apply `accLen` to 10, and display the size of the resulting set of strings, as well as the set of strings itself. Include a transcript of your Forlan session. Your definition of `accLen` should reside in the file `ps4-p2.sml` of the subdirectory `CS516-PS4` of your private GitHub repository. [10 points]

## Problem 3 (45 points)

Let the finite automaton  $M$  be as in Problem 2. Define a function  $\mathbf{diff} \in \{0,1\}^* \rightarrow \mathbb{Z}$  by: for all  $w \in \{0,1\}^*$ ,

$$\mathbf{diff} w = \text{the number of 1's in } w - \text{the number of 0's in } w.$$

Let

$$X = \{w \in \{0,1\}^* \mid \text{for all substrings } v \text{ of } w, -2 \leq \mathbf{diff} v \leq 2\}.$$

Prove that  $L(M) = X$ .

Hint: you may find it useful to study the model answer to Problem 3 of the old Problem Set 4. Your proof should *not* rely on the correctness of Forlan.