

Problem Set 7

Due by 11:59pm on Wednesday, May 4
Submission via Gradescope and GitHub

Problem 1 (20 points)

Let $X = \{0^i 1^j 2^k 3^l \mid i, j, k, l \in \mathbb{N} \text{ and } i = k \text{ and } j = l\}$. Thus $011233 = 0^1 1^2 2^1 3^2 \in X$. On the other hand, $01133 = 0^1 1^2 2^0 3^2 \notin X$ and $0112333 = 0^1 1^2 2^1 3^3 \notin X$.

Prove that X is not context-free.

Problem 2 (15 points)

Consider the proof—in Section 4.10 of the lecture slides and/or the book—of the Pumping Lemma for Context-free Languages. When looking for the first repetition of variables in the path pat through the parse tree pt , would the proof still work if we searched forwards from the start of the path (from the top of the tree, down), instead of backwards from the end of the path (from the bottom of the tree, up)?

Problem 3 (45 points)

Let $X = \{0^i 1^j 2^k 3^l \mid i, j, k, l \in \mathbb{N} \text{ and } i + j \leq k + l\}$, and let the grammar G be

$A \rightarrow 0A3 \mid A3 \mid B \mid C$

$B \rightarrow 1B3 \mid D$

$C \rightarrow 0C2 \mid D$

$D \rightarrow \% \mid 1D2 \mid D2$

Prove that $L(G) = X$.

Problem 4 (20 points)

Let $Y = \{0^i 1^j 2^k 3^l \mid i, j, k, l \in \mathbb{N} \text{ and } i + j \leq k + l \text{ and } i + j + k + l \text{ is odd}\}$. In other words, the elements of Y are the elements of the language X of Problem 3 whose lengths are odd numbers. For example, $01223 \in X$, but $0123 \notin X$.

Find a grammar H such that $L(H) = Y$. Try to make H as easy to understand as possible.

Put the expression of H in Forlan's syntax in the file `ps7-p4-gram` of the subdirectory `CS516-PS7` of your private GitHub repository. You *don't* have to test your definition of H , although that is one way of giving you high-confidence in the correctness of your answer.

Hint: one approach to finding H is to:

- use Forlan to turn the grammar G of Problem 3 into a grammar generating Y ;
- use Forlan's hand-simplification functions to make this grammar easier to understand; and
- use this hand-simplified grammar as the inspiration for your definition of H .

If you want to follow this approach and show these steps as part of your answer, please do. But this is not required.