CS 516—Software Foundations via Formal Languages—Spring 2025

Problem Set 1

Due by 11:59pm on Wednesday, February 5
Submission via Gradescope

Problem 1 (25 points)

(a) Either prove or disprove the following statement:

For all sets A, B and C,

$$A - (B \cup C) = (A - B) - C.$$

[12 points]

(b) Either prove or disprove the following statement:

For all sets A, B and C,

$$A - (B \cap C) = (A - B) \cup (A - C).$$

[13 points]

Problem 2 (25 points)

Define a function $f \in \mathbb{N} \to \mathbb{N}$ by recursion:

$$f\,0 = 0,$$
$$f(n+1) = f\,n + n.$$

Use the Principle of Mathematical Induction to show that, for all $n \in \mathbb{N}$,

$$2(f\,n) = n^2 - n.$$

Problem 3 (25 points)

Use the Principle of Strong Induction to prove that, for all $n \in \mathbb{N}$, if $n \geq 18$, then there are $i, j \in \mathbb{N}$ such that $n = 4i + 7j$.

Problem 4 (25 points)

Suppose A is a set, R is a relation on A, and R is *not* well-founded on A. Suppose we were allowed to use Theorem 1.2.8 (Principle of Well-founded Induction) with R (even though it is not well-founded). Prove an obviously false statement; try to make your statement as simple as possible.