

Problem Set 4

Due by 11:59pm on Monday, March 24

Problem 1 (35 points)

Let

$A = \{001, 011, 101, 111\}$, and

$B = \{w \in \{0, 1\}^* \mid \text{for all } x, y \in \{0, 1\}^*, \text{ if } w = x0y,$

then there is a $z \in A$ such that z is a prefix of $y\}$.

(a) Find and draw (e.g., using JForlan) a finite automaton M such that $L(M) = B$. Try to make M have as few states as possible. [20 points]

(b) Define an SML/Forlan function

```
val test = fn : int -> fa -> str option * str option
```

such that, for all $n \in \mathbb{N}$, `test n` returns a function f such that, for all FAs N , $f N$ returns a pair $(xOpt, yOpt)$ such that:

- If there is an element of $\{0, 1\}^*$ of length no more than n that is in B but is not accepted by N , then $xOpt = \text{SOME } x$ for some such x ; otherwise, $xOpt = \text{NONE}$.
- If there is an element of $\{0, 1\}^*$ of length no more than n that is not in B but is accepted by N , then $yOpt = \text{SOME } y$ for some such y ; otherwise, $yOpt = \text{NONE}$.

Use `test` to test your FA M from part (a) on all elements of $\{0, 1\}^*$ of length no more than 20. Include a transcript of your Forlan session in your PDF submission.

In the subdirectory `CS516-PS4` of your private GitHub repository you should put:

- a file `ps4-p1-fa` containing the expression in Forlan's syntax of M ;
- a file `ps4-p1.sml` consisting of the definition of `test`.

Hint: you may adapt code from the file `ps4-p1.sml` from Problem 1 of the old Problem Set 4. [15 points]

Problem 2 (65 points)

Let the languages A and B be as in Problem 1, and let the finite automaton M be your solution to Problem 1(a).

(a) Use strong string induction to prove that $B \subseteq L(M)$. [32 points]

(b) Use induction on Λ to prove that $L(M) \subseteq B$. [33 points]