CS 516—Software Foundations via Formal Languages—Spring 2025

Problem Set 5

Due by 11:59pm on Thursday, April 3

Problem 1 (60 points)

(a) Define functions $\mathbf{zo}, \mathbf{oz} \in \{0,1\}^* \to \mathbb{N}$ by recursion:

- $\mathbf{zo}\,\% = 0$,
- for all $w \in \{0,1\}^*$,

$$\mathbf{zo}(w1) = \begin{cases} 0, & \text{if } w = \%, \\ \mathbf{zo}\,w, & \text{if 1 is a suffix of } w, \\ \mathbf{zo}\,w + 1, & \text{if 0 is a suffix of } w, \end{cases}$$

- for all $w \in \{0,1\}^*$, $\mathbf{zo}(w0) = \mathbf{zo}\,w$;

and

- $\mathbf{oz}\,\% = 0$,
- for all $w \in \{0,1\}^*$,

$$\mathbf{oz}(w0) = \begin{cases} 0, & \text{if } w = \%, \\ \mathbf{oz}\,w, & \text{if 0 is a suffix of } w, \\ \mathbf{oz}\,w + 1, & \text{if 1 is a suffix of } w, \end{cases}$$

- for all $w \in \{0,1\}^*$, $\mathbf{oz}(w1) = \mathbf{oz}\,w$.

(Informally, $\mathbf{zo}\,w$ is the number of occurrences of 01 in w, and $\mathbf{oz}\,w$ is the number of occurrences of 10 in w. E.g., $\mathbf{zo}(1001) = \mathbf{oz}(1001) = 1$, $\mathbf{zo}(0101) = 2$ and $\mathbf{oz}(0101) = 1$.)

Define

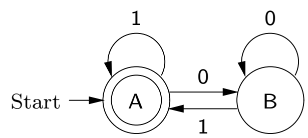
$$X = \{\, w \in \{0,1\}^* \mid \mathbf{zo}\,w \text{ is even or } \mathbf{oz}\,w \text{ is even} \,\}.$$

Find and draw (e.g., using JForlan) a DFA M such that $L(M) = X$. Hint: you may use Forlan to test your definition of M, but this is not required. [15 points]

(b) Prove that your answer to part (a) is correct using our standard method of proving the correctness of DFAs (induction on Λ plus proof by contradiction). Your proof must explicitly use the definitions of $\mathbf{zo}$ and $\mathbf{oz}$ (and must not use the informal concepts of numbers of occurrences of 01 and 10). [45 points]

Problem 2 (20 points)

Let N be the DFA

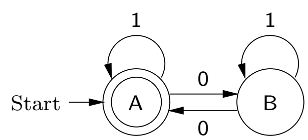


Use Forlan to convert N into a regular expression α such that $L(\alpha) = L(N)$. Try to make α be as simple as possible. Include a transcript of your Forlan session.

Problem 3 (20 points)

Suppose M is a DFA. We say that M is *reversible* iff, for all $r \in Q_M$ and $a \in \textbf{alphabet}(M)$, there is a unique $q \in Q_M$ such that $q, a \to r \in T_M$.

For example,



is reversible. On the other hand the DFA N of Problem 2 is not reversible, because both $\mathsf{B}, 1 \to \mathsf{A}$ and $\mathsf{A}, 1 \to \mathsf{A}$ are transitions labeled by 1 leading into A. (And also because both $\mathsf{A}, 0 \to \mathsf{B}$ and $\mathsf{B}, 0 \to \mathsf{B}$ are transitions labeled by 0 leading into B.)

Either prove or disprove the following statement:

For all regular languages L, there is a reversible DFA M such that $L(M) = L$.